

Exercice 1

$$A = 2^9 \times 2^{-4} = 2^{9-4} = 2^5 = 32 \qquad B = \frac{3^{-7}}{3^{-9}} = 3^{-7-(-9)} = 3^{-7+9} = 3^2 = 9$$

$$C = \frac{(-4)^1}{(-4)^{-2}} = (-4)^{1-(-2)} = (-4)^{1+2} = (-4)^3 = -64 \qquad D = \frac{5^{-3} \times 5^{-2}}{5^{-6}} = \frac{5^{-5}}{5^{-6}} = 5^{-5-(-6)} = 5^{-5+6} = 5^1 = 5$$

$$E = \frac{(5-3 \times 2)^4}{(4-5)^{-5}} = \frac{(5-6)^4}{(-1)^{-5}} = \frac{(-1)^4}{(-1)^{-5}} = (-1)^{4-(-5)} = (-1)^9 = -1$$

$$F = \frac{12 \times 10^5 \times 5 \times 10^7}{15 \times 10^4 \times 2 \times 10^{-3}} = \frac{2 \times 6 \times 5}{5 \times 3 \times 2} \times \frac{10^7 \times 10^5}{10^4 \times 10^{-3}} = 2 \times \frac{10^{12}}{10^1} = 2 \times 10^{11}$$

Exercice 2

$$A = \frac{3^{-4} \times 3^8}{3^5 \times 3^{-7}} = \frac{3^4}{3^{-2}} = 3^{4-(-2)} = 3^{4+2} = 3^6 = 729 \qquad B = \frac{(-3)^{-4} \times (-3)^0}{(-3)^{-4}} = \frac{(-3)^{-4}}{(-3)^{-4}} = 1$$

$$C = \left(\frac{11^{-11} \times 11^9}{11^{-13} \times 11^{12}} \right)^2 = \left(\frac{11^{-2}}{11^{-1}} \right)^2 = (11^{-2-(-1)})^2 = (11^{-1})^2 = 11^{-2} = \frac{1}{11^2} = \frac{1}{121}$$

Exercice 3

$$A = \frac{10^9}{10^4 \times 10^3} = \frac{10^9}{10^7} = 10^{9-7} = 10^2 \qquad B = \frac{10^{-8} \times 10^9}{10^{-3}} = \frac{10^1}{10^{-3}} = 10^{1-(-3)} = 10^{1+3} = 10^4$$

$$C = \frac{(10^{-3})^{-2}}{10^{19}} = \frac{10^6}{10^{19}} = 10^{6-19} = 10^{-13}$$

Exercice 4

$$A = \left(\frac{a}{b} \right)^5 = \frac{a^5}{b^5} = a^5 \times b^{-5} \qquad B = \frac{a^7}{a^3 \times b^{-5}} = a^7 \times a^{-3} \times b^5 = a^4 \times b^5$$

$$C = \left(\frac{b}{a} \right)^7 \times a^3 = b^7 \times a^{-7} \times a^3 = a^{-4} \times b^7$$

Exercice 5

$$A = 2^3 \times (23 - 5^2)^{-3} \times (2 - 3)^5 = 2^3 \times (23 - 25)^{-3} \times (-1)^5 = 8 \times (-2)^{-3} \times (-1)$$

$$A = 8 \times \frac{1}{(-2)^3} \times (-1) = 8 \times \frac{1}{-8} \times (-1) = 1$$

$$B = 3^{-2} \times (13 - 2^2)^2 \times (6^2 - 33)^{-2} = 3^{-2} \times (13 - 4)^2 \times (36 - 33)^{-2} = 3^{-2} \times 9^2 \times 3^{-2}$$

$$B = 3^{-2} \times (3^2)^2 \times 3^{-2} = 3^{-2} \times 3^4 \times 3^{-2} = 3^0 = 1$$

Exercice 6

$$A = \frac{25 \times 10^{-3} \times 3 \times 10^9}{15 \times (10^2)^{-4}} = \frac{25 \times 3}{15} \times \frac{10^{-3} \times 10^9}{(10^2)^{-4}} = \frac{5 \times 5 \times 3}{5 \times 3} \times \frac{10^{-3} \times 10^9}{10^{-8}}$$

$$A = \frac{5 \times 10^6}{10^{-8}} = 5 \times 10^{6 - (-8)} = 5 \times 10^{14}$$

Exercice 7

$$A = \frac{9^{-3} \times 81^2}{27^3} = \frac{(3^2)^{-3} \times (3^4)^2}{(3^3)^3} = \frac{3^{-6} \times 3^8}{3^9} = \frac{3^2}{3^9} = 3^{2-9} = 3^{-7}$$

$$B = 9^{-5} \times 27^3 = (3^2)^{-5} \times (3^3)^3 = 3^{-10} \times 3^9 = 3^{-1}$$

$$C = \frac{81^3 \times 9^{-5}}{3^{-1}} = \frac{(3^4)^3 \times (3^2)^{-5}}{3^{-1}} = \frac{3^{12} \times 3^{-10}}{3^{-1}} = \frac{3^2}{3^{-1}} = 3^{2 - (-1)} = 3^{2+1} = 3^3$$

Exercice 8

$$8^{2^3} = 8^8 = (2^3)^8 = 2^{24} \text{ et } (8^2)^3 = 64^3 = (2^6)^3 = 2^{18} \text{ donc } 8^{2^3} \neq (8^2)^3$$

Exercice 9

$$2^{19} = 2^4 \times 2^{15} \text{ donc } 2^{19} \times 5^{15} = 2^4 \times 2^{15} \times 5^{15} = 2^4 \times 10^{15} = 16 \times 10^{15}$$

Exercice 10

1. Il y a $10 \times 10 \times 10 \times 10 = 10^4 = 10\,000$ codes possibles.
2. Il faut une minute pour tester 10 codes donc il faut 1000 minutes pour tester 10 000 codes.
Or $1000 = 60 \times 16 + 40$ donc il faut 16h40min pour tester 10 000 codes.

Exercice 11

$$A = t^4 \times (t^3)^2 \times t^7 = t^4 \times t^6 \times t^7 = t^{17}$$

$$B = \frac{t^{19}}{t^8 \times t^{-3} \times (t^{-1})^{-1}} = \frac{t^{19}}{t^8 \times t^{-3} \times t^1} = \frac{t^{19}}{t^6} = t^{19-6} = t^{13}$$

Exercice 12

$$C = (a^{-3} \times b^2)^{-8} = (a^{-3})^{-8} \times (b^2)^{-8} = a^{24} \times b^{-16} \quad D = \left(\frac{a}{b}\right)^{-5} \times b^6 = \frac{a^{-5}}{b^{-5}} \times b^6 = a^{-5} \times b^5 \times b^6 = a^{-5} \times b^{11}$$

Exercice 13

$$E = 2^n \times 2^{n-3} = 2^{n+n-3} = 2^{2n-3} \quad F = \frac{2^{4-n} \times 2^{n+5}}{2^{-7n}} = \frac{2^{4-n+n+5}}{2^{-7n}} = \frac{2^9}{2^{-7n}} = 2^{9-(-7n)} = 2^{7n+9}$$

Exercice 14

$$A = 45 \times 10^{12} \times 4 \times 10^{-26} = 45 \times 4 \times 10^{-14} = 180 \times 10^{-14} = 1,8 \times 10^2 \times 10^{-14} = 1,8 \times 10^{-12}$$

$$B = 7,35 \times 10^{-12} + 0,045 \times 10^{-11} - 40 \times 10^{-14} = 10^{-12} \times (7,35 + 0,045 \times 10^1 - 40 \times 10^{-2})$$

$$B = 10^{-12} \times (7,35 + 0,45 - 0,4) = 7,4 \times 10^{-12}$$

$$C = 3500000000000^2 = (3,5 \times 10^{12})^2 = (3,5)^2 \times (10^{12})^2 = 12,25 \times 10^{24} = 1,225 \times 10^{25}$$

$$D = \frac{49 \times 10^{-5} \times 6 \times 10^7}{3 \times 10^3 \times 7 \times 10^{-3}} = \frac{7 \times 7 \times 3 \times 2}{3 \times 7} \times \frac{10^{-5} \times 10^7}{10^3 \times 10^{-3}} = 14 \times \frac{10^2}{10^0} = 14 \times 10^2 = 1,4 \times 10 \times 10^2 = 1,4 \times 10^3$$

Exercice 15

$$1. \quad A = 2^{31} + 2^{30} + 2^{29} = 2^1 \times 2^{30} + 2^{30} + 2^{-1} \times 2^{30} = (2 + 1 + 0,5) \times 2^{30} = 3,5 \times 2^{30}$$

$$2. \quad B = 4^{15} = (2^2)^{15} = 2^{30}$$

$$3. \quad \frac{A}{x} = \frac{B}{2} \Leftrightarrow 2A = Bx \Leftrightarrow x = \frac{2A}{B} = \frac{2 \times 3,5 \times 2^{30}}{2^{30}} = 7$$

Exercice 16

$$1. \quad \text{Nombre de soldats} = 1 \times 5 \times 5 \times 5 \times 5 \times 5^2 = 5^6$$

$$2. \quad \text{Effectif total} = 1 + 5^6 = 1 + 15625 = 15626 \quad (\text{on ne compte pas la Reine})$$

$$3. \quad \text{On a } 10^4 = 10000 \text{ et } 10^5 = 100000 \text{ donc la Reine des termites devra lever une armée d'effectif égal à } 10^5.$$

Exercice 17

$$A = 10^5 \times (10^{-3})^4 = 10^5 \times 10^{-12} = 10^{-7} \quad B = 10 \times (10^{-7})^3 \times 10^9 = 10^{1-21+9} = 10^{-11}$$

$$C = 2^3 \times 5^3 \times 10^8 = (2 \times 5)^3 \times 10^8 = 10^3 \times 10^8 = 10^{11} \quad D = \frac{10^{-2} \times 10^{-7}}{10^6} = \frac{10^{-9}}{10^6} = 10^{-9-6} = 10^{-15}$$

$$E = \frac{10^{-4} \times 10^9}{10^5 \times 10^{-7}} = \frac{10^5}{10^{-2}} = 10^{5-(-2)} = 10^{5+2} = 10^7$$

$$F = \frac{(10^4)^{-2} \times 10}{10^{-3}} = \frac{10^{-8} \times 10^1}{10^{-3}} = \frac{10^{-7}}{10^{-3}} = 10^{-7-(-3)} = 10^{-7+3} = 10^{-4}$$

$$G = \left(\frac{10^{13} \times 10^{-9}}{10^{-14} \times 10^{-8}} \right)^2 = \left(\frac{10^4}{10^{-22}} \right)^2 = (10^{4-(-22)})^2 = (10^{26})^2 = 10^{52}$$

$$H = \frac{20^6 \times 10^{-9}}{2^6} = \frac{(2 \times 10)^6 \times 10^{-9}}{2^6} = \frac{2^6 \times 10^6 \times 10^{-9}}{2^6} = 10^{-3}$$

Exercice 18

$$\begin{aligned} 1. \quad a.l &= 3 \times 10^5 \times 365 \times 24 \times 3600 = 3 \times 10^5 \times 3,65 \times 10^2 \times 2,4 \times 10^1 \times 3,6 \times 10^3 \\ a.l &= 3 \times 3,65 \times 2,4 \times 3,6 \times 10^{11} = 94,608 \times 10^{11} = 9,4608 \times 10^{12} \end{aligned}$$

$$\begin{aligned} 2. \quad 5000 \text{ ans} &= 5000 \times 365 \times 24 \times 3600 = 5 \times 10^3 \times 3,65 \times 10^2 \times 2,4 \times 10^1 \times 3,6 \times 10^3 \\ 5000 \text{ ans} &= 5 \times 3,65 \times 2,4 \times 3,6 \times 10^9 = 157,68 \times 10^9 = 1,5768 \times 10^{11} \\ \text{distance en km} &= 3 \times 10^5 \times 1,5768 \times 10^{11} = 4,7304 \times 10^{16} \text{ km} \end{aligned}$$

Exercice 19

$$\begin{aligned} 1 \text{ livre} &= 273 \times 10^{24} u \\ 1 \text{ kg} &= 0,0602 \times 10^{28} u = 0,0602 \times 10^4 \times 10^{24} u = 602 \times 10^{24} u \\ 1 \text{ kann} &= 22,6 \times 10^{26} u = 22,6 \times 10^2 \times 10^{24} u = 22600 \times 10^{24} u \\ 1 \text{ tael} &= 2,28 \times 10^{25} u = 2,28 \times 10^1 \times 10^{24} u = 22,8 \times 10^{24} u \end{aligned}$$

Conclusion : $1 \text{ tael} < 1 \text{ livre} < 1 \text{ kg} < 1 \text{ kann}$