

Exercice 1

$$z_1 = 2 \Leftrightarrow \operatorname{Re}(z_1) = 2 \text{ et } \operatorname{Im}(z_1) = 0$$

$$z_2 = -3i \Leftrightarrow \operatorname{Re}(z_2) = 0 \text{ et } \operatorname{Im}(z_2) = -3$$

$$z_3 = i - 3 = -3 + i \Leftrightarrow \operatorname{Re}(z_3) = -3 \text{ et } \operatorname{Im}(z_3) = 1$$

$$z_4 = z_1 + z_3 = 2 + i - 3 = -1 + i \Leftrightarrow \operatorname{Re}(z_4) = -1 \text{ et } \operatorname{Im}(z_4) = 1$$

$$z_5 = z_2 \times z_3 = -3i \times (i - 3) = -3i^2 + 9i = 3 + 9i \Leftrightarrow \operatorname{Re}(z_5) = 3 \text{ et } \operatorname{Im}(z_5) = 9$$

Exercice 2

$$a + 3i = 2 + i(1 - b) \Leftrightarrow \begin{cases} a = 2 \\ 1 - b = 3 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = -2 \end{cases}$$

$$2 + a + i(b^2 + b) = i(2b - ia^2) + 3a + 3 \Leftrightarrow (2 + a) + i(b^2 + b) = 2ib + a^2 + 3a + 3$$

$$2 + a + i(b^2 + b) = i(2b - ia^2) + 3a + 3 \Leftrightarrow (2 + a) + i(b^2 + b) = (a^2 + 3a + 3) + 2ib$$

$$2 + a + i(b^2 + b) = i(2b - ia^2) + 3a + 3 \Leftrightarrow \begin{cases} a^2 + 3a + 3 = 2 + a \\ b^2 + b = 2b \end{cases} \Leftrightarrow \begin{cases} a^2 + 2a + 1 = 0 \\ b^2 - b = 0 \end{cases} \Leftrightarrow \begin{cases} (a+1)^2 = 0 \\ b(b-1) = 0 \end{cases}$$

$$2 + a + i(b^2 + b) = i(2b - ia^2) + 3a + 3 \Leftrightarrow \begin{cases} a = -1 \\ b = 0 \text{ ou } b = 1 \end{cases}$$

Conclusion : $S = \{(-1; 0) ; (-1; 1)\}$

Exercice 3

- $z_1 = (3 - 2i) - (3 + 2i) = 3 - 2i - 3 - 2i = -4i$
- $z_2 = 2(1 + i) + i(2i - 1) = 2 + 2i - 2 - i = i$
- $z_3 = (1 + i)(3 + 2i) = 3 + 2i + 3i - 2 = 1 + 5i$
- $z_4 = (1 - i)^5 = 1^5 - 5 \times 1^4 \times i^1 + 10 \times 1^3 \times i^2 - 10 \times 1^2 \times i^3 + 5 \times 1^1 \times i^4 - i^5$
 $z_4 = (1 - i)^5 = 1 - 5i - 10 + 10i + 5 - i = -4 + 4i$

Rappel : triangle de Pascal pour $n=5$

$n = 0$	1					
$n = 1$	1	1				
$n = 2$	1	2	1			
$n = 3$	1	3	3	1		
$n = 4$	1	4	6	4	1	
$n = 5$	1	5	10	10	5	1
	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$

Exercice 4

$$z_1 = -\left(2 - \frac{1}{4}i\right) + \left(\frac{1}{3}i - 1\right) = -2 + \frac{1}{4}i + \frac{1}{3}i - 1 = -3 + \frac{7}{12}i$$

$$z_2 = \left(\frac{1}{3} + \frac{3}{4}i\right)\left(\frac{2}{3}i - \frac{1}{4}\right) = \frac{2}{9}i - \frac{1}{12} - \frac{1}{2} - \frac{3}{16}i = \frac{-7}{12} + \frac{5}{144}i$$

$$z_3 = \left(2\sqrt{3} - i\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2} + 2i\sqrt{2}\right) = 2\sqrt{3} - i\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} - 2i\sqrt{2} = \frac{3\sqrt{3}}{2} - \frac{5\sqrt{2}}{2}i$$

$$z_4 = \left(i\frac{\sqrt{3}}{4} + \frac{\sqrt{2}}{2}\right)\left(-i\frac{\sqrt{3}}{4} + \frac{\sqrt{2}}{2}\right) = \frac{3}{16} + \frac{\sqrt{6}}{8}i - \frac{\sqrt{6}}{8}i + \frac{1}{2} = \frac{11}{16}$$

Exercice 5

$$1. \quad i^2 = -1; i^3 = i^2 \times i = -i; i^4 = i^3 \times i = -i \times i = 1 \text{ et } i^5 = i^4 \times i = i$$

$$2. \quad (a) \quad i^{4k} = i^{4k} = 1^k = 1$$

$$i^{4k+1} = i^{4k} \times i^1 = 1 \times i = i$$

$$i^{4k+2} = i^{4k+1} \times i = i \times i = -1$$

$$i^{4k+3} = i^{4k+2} \times i^1 = -1 \times i = -i$$

$$(b) \quad Z = i^{2020} + i^{2021} + i^{2022} + i^{2023} = i^{4 \times 505} + i^{4 \times 505 + 1} + i^{4 \times 505 + 2} + i^{4 \times 505 + 3} = 1 + i - 1 - i = 0$$

$$3. \quad Z_n = i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1 + i + i^2 + i^3) = i^n(1 + i - 1 - i) = i^n \times 0 = 0$$

Exercice 6

$$1. \quad i^2 = -1 < 0 \text{ donc la proposition 1 est fausse.}$$

$$2. \quad (1+i)(3+3i) = 3+3i+3i-3 = 6i \text{ donc la proposition 2 est vraie.}$$

$$3. \quad z = (2i-1)^2 + 2(2i-1) + 5 = (2i)^2 - 2 \times (2i) \times 1 + 1^2 + 4i - 2 + 5$$

$$z = -4 - 4i + 1 + 4i - 2 + 5 = 0 \text{ donc la proposition 3 est vraie.}$$

$$4. \quad z = (2-i\sqrt{3})^2 + 4(2-i\sqrt{3}) + 7 = 4 - 4\sqrt{3}i - 3 + 8 - 4\sqrt{3}i + 7$$

$$z = 16 - 8\sqrt{3}i \neq 0 \text{ donc la proposition 4 est fausse.}$$